3 The Qualitative Analysis of Beams

In the next two chapters, you will study an approach to the analysis of structures that is likely to be unfamiliar to you. The traditional approach to the study of structural analysis has been based, almost exclusively, on quantitative (i.e. numerical) methods. On one hand, this is entirely logical. The structural designer does, eventually, need to put values to loads and dimensions and determine the numerical value of reactions, bending moments and so on.

There is, however, a critically important stage before that numerical analysis; the preliminary analysis required to size the structural members. The nature of the analysis of statically indeterminate structures is such that the designer must know the size of *all* the structural members *before* an analysis is carried out. Consequently, it should be clear that the detailed, numerical analysis of the structure, nowadays carried out by computer, is a check on that preliminary analysis.

This places great importance on that stage of the design procedure. If the structure is incorrectly sized, repeated analysis will need to be carried out. This is true regardless of whether the analysis is being carried out by hand *or* by computer.

It is generally assumed that the skills required for this preliminary analysis are the inevitable consequence of studying numerical techniques. The results of extensive research into the development of an understanding of structural behaviour (See Appendix: DMB papers) in undergraduates and trainee engineers suggest very strongly that this is not the case.

That preliminary analysis requires a quite different set of skills that are referred to here as ‘qualitative’. Essentially non-numerical, the basis of these skills is the recognition of the relationship between the load and the resulting behaviour of the structure. In simple structures, that is the relationship between the load, deflected shape and the resulting reactions. These skills are distinctly different from quantitative skills; they rely on sets of coherent diagrams rather than mathematical models, for example. However, the most obvious difference between the qualitative and the quantitative approach to the analysis of

structures is that there is no obvious sequence for the steps in the qualitative approach.

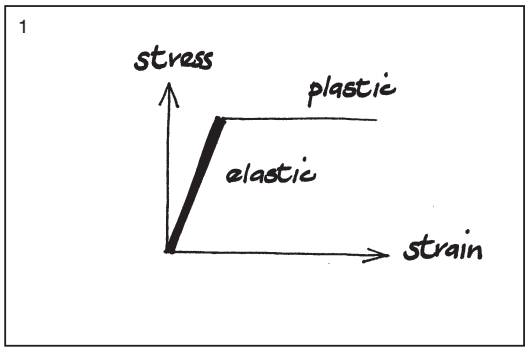
Once a particular numerically based analytical technique has been learned, then the sequence of the solution will almost always be the same. However, faced with a qualitative analysis it is not apparent from which point the solution will emerge most effectively. It may be best to start with the bending moment diagram in one case or the deflected shape in another.

In many structural problems it is necessary to start with all parts of the full solution at the same time, which is potentially very confusing. Students studying this approach say that they understand each of the steps in the examples then, as soon as they try one of the practice problems they are stuck. This suggests that the teaching method is, in some way, deficient. This is not necessarily so. The explanation is that qualitative problems are *inherently* difficult. This new approach requires a shift of attitude and this takes time and practice.

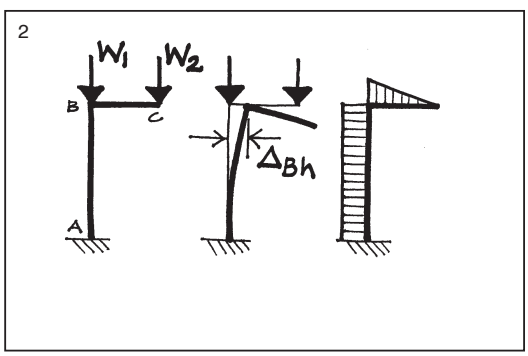
However, you will discover as you begin to grasp this approach, that you find a significantly increased confidence in your *overall* understanding of structural behaviour that will contribute to your understanding of problems of structural design and the methods of analysis studied in later chapters.

Perhaps the greatest value of this qualitative approach will only be apparent to you once you start your office training. The use of computers is now universal. How can you check that the output is correct? The checking system must be independent and robust. You will discover that a sound understanding of structural behaviour will play a significant part in the overall checking procedures that must be constructed to ensure the correct use of the computer and the elimination of data and modelling errors.

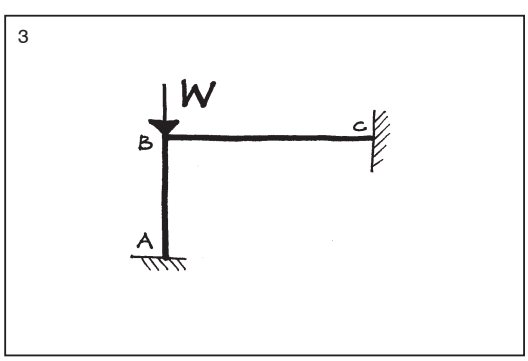
1 We will start by restating the assumptions that underlie the elastic analysis of structures. The first is that, for all loading cases, the structure is within the elastic range of material behaviour. Stress is directly proportional to strain and deflections directly proportional to load.



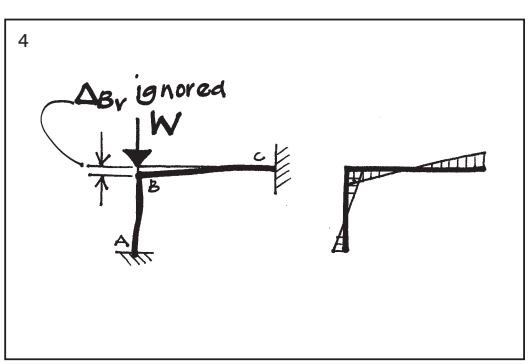
2 The second assumption is that deformations due to the loading do not create secondary bending moments. The cantilever bent ABC is subject to two point loads W_1 and W_2. Here the load W_2 will cause bending in the cantilever BC that will result in a horizontal deflection at B, Δ_{Bh}. The secondary moment $W_1 \times A_{Bh}$ is ignored. In the Engineer's Theory of Bending it is assumed that deformations due to loading do not result in significant changes in the structural geometry.

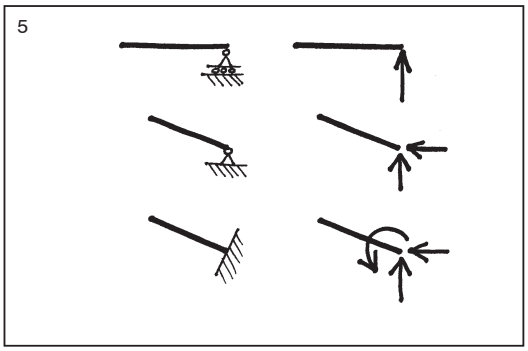


3 In the *qualitative* analysis of structures and most manual numerical analysis, axial loading, which would result in axial strain in the member, is ignored in the determination of bending moments. The frame, ABC is loaded in the vertical direction at B.

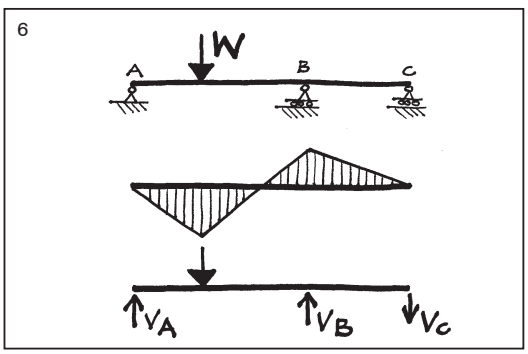


4 Although the load W would cause the column AB to shorten, it is assumed in the qualitative analysis that there is no secondary moment in beam BC. Generally, in the elastic analysis of structures, deformations are considered to be small compared with the dimensions of the structure so that the equations of equilibrium of the deformed structure are consistent with those of the un-deformed structure. It should be noted however, that this deformation A_{Bv} is included in the computer analysis of structures.

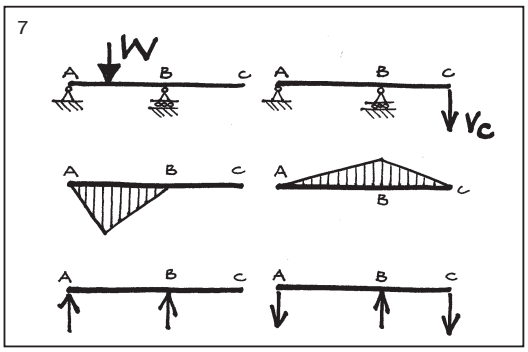




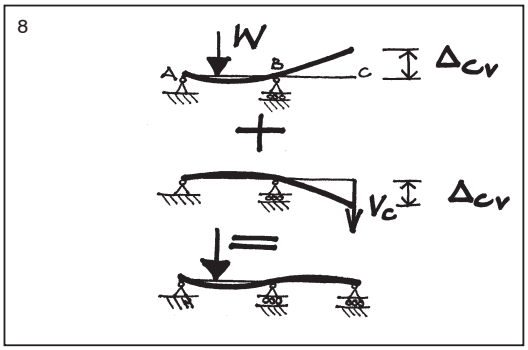
5 It is important to restate the diagrammatic convention for the support conditions and the resulting reactions. The roller support has one reaction, normal to the direction of movement. The pinned support has two force reactions and the fully fixed support two force and one moment reaction.



6 The principle of superposition is assumed to apply as the structures are within the elastic range. We will illustrate this principle with this example. The load W on the two-span beam ABC produces a hogging moment over support B with a downward reaction V_C, to balance this bending moment at C.



7 If we remove the vertical reaction at C, the beam becomes a simply supported beam with cantilever. If we now apply the reaction at C, V_C as a load, the combination of these two conditions will produce the solution to the original two-span beam. This simple example illustrates one of the most powerful design tools in the armoury of the designer, since it allows the reduction of complex structures into simpler forms and the examination of the load effects separately.



8 The deflections may be treated in the same way. The vertical deflection at C due to the load W must be equal to the downward deflection at C due to the force reaction V_C since the real deflection at C is zero.

9 This is the first example of the qualitative analysis of structures, the simply supported beam ABC, with a cantilever CD. The full solution of a qualitative analysis must always consist of:

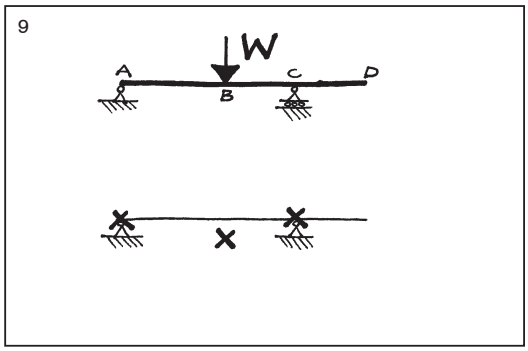
the deflected shape

reactions

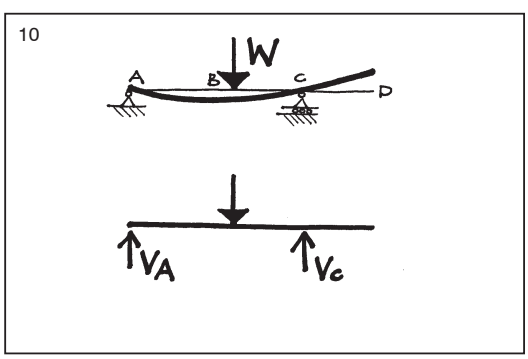
bending moment diagram

You must become accustomed to the idea that they are of equal importance and your qualitative analysis will not be correct until each of them has been determined and found to check against the other two.

In the qualitative analysis of beams, the best place to start is likely to be the deflected shape. We will start the deflected shape by identifying the points through which we are certain that the deflected beam must pass, A and C and a downwards deflection beneath the load.

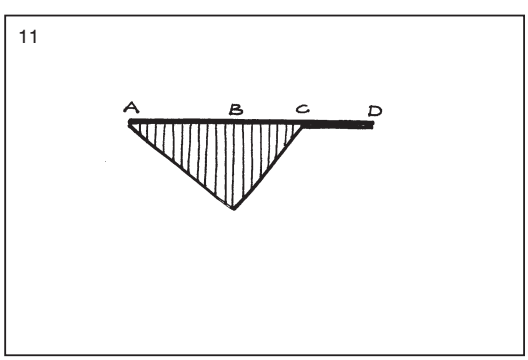


10 We can now draw the smooth elastic curve through these three points. Note that CD is straight. The curve must be smooth and with practice a very accurate deflected shape may be drawn. Try practising the identification of the deflected shape with a flexible plastic ruler.



11 This simple example of the bending moment diagram illustrates the need to distinguish between the structure, which is drawn as the base line of the bending moment diagram, and the diagram itself. The point here is that the value of the bending moment over the portion of the span CD is zero.

N.B. The bending moment diagram is always drawn on the bending tension side of the structure.



12

* If a part of the structure remains straight after loading — NO BENDING MOMENT

12 There are certain simple rules that should be borne in mind when carrying out the qualitative analysis. Experience suggests that because this approach is unfamiliar, students tend to become confused, reach a solution containing obvious incompatibilities and are not able to spot them. Having found a solution consisting of the deflected shape, reactions and bending moment diagram you should take a cool look at each part and see how it relates to the other. Is there a bending moment at the pinned support? Unlikely. Is the bending moment at an internal support zero? Very unlikely. Does the tension identified by the deflected shape agree with the bending moment diagram drawn on the tension side of the structure? We will call them *golden rules*.

Here is the first.

13

* If the loading on a structure consists only of point loads, then the BENDING MOMENT DIAGRAM will always consist of STRAIGHT LINES.

13 And here is another.

Bending moments resulting from point loads are linear expressions, therefore the bending moment diagram consists of straight lines. In addition the diagram can only change direction at a load or reaction.

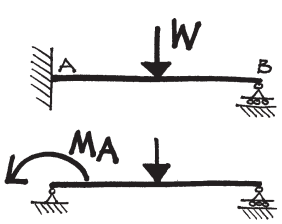
14

* The bending moment is always zero at a simple support

14 And another.

That is, where the hinge occurs at the end of a structure, or at an internal hinge which we will study later in this chapter.

15



15 We will now carry out a qualitative analysis on the propped cantilever AB. We will apply the principle of superposition to examine the effect of the fully fixed support at A, by the notional removal of the moment reaction at A, M_A .